Quadratic Functions

Quadratic functions are functions that can be written in the form $f(x) = ax^2 + bx + c$, where a, b and c are constants (and $a \neq 0$).

1. Which of the following are quadratic functions?

$$x^2 \qquad 3 - \frac{x^2}{6} \qquad 1 + \frac{5}{x^2} \qquad 3(x+2)^2 + x \qquad x(x+2) \qquad \pi x^2$$

2. Math's always full of silly rules. Why do we insist that a not be zero in the definition?

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3. Hopefully you've learned it before. What's the Quadratic Formula?

4. What's the Quadratic Formula good for and when do we need it?

5. Let $f(x) = x^2 - 3x - 4$.
   (a) Can you factor $f(x)$?

   (b) What are the roots of f?

6. Let $f(x) = x^2 - 3x - 3$.
   (a) What are the roots of f? (Hint: if you can't factor it, you can use the...)

   (b) Can you factor $f(x)$?

7. Does every quadratic function have roots? If not, how can you tell if one has roots or not?

The roots of a quadratic function (if they exist) can tell us something about the graph of the function.

8. If you know the roots of a quadratic function, what can you say about what its graph looks like? Can you describe it completely?

9. What if a quadratic function doesn't have roots? What, if anything, could you say about its graph?

10. If you also knew the vertex, would that be helpful?

11. Can you say easily what the vertex of $f(x) = x^2 - 3x - 3$ is? (Let's say you have 5 seconds to say what the vertex is, and if you get it you win \$1 million. Can you do it?)

The two main forms in which we deal with quadratic functions are the form $ax^2 + bx + c$, and the form $p(x+q)^2 + r$. Sometimes these have names, like "standard form," but we'll avoid names since those aren't universal. Since both forms are useful, it's helpful to know how to go back and forth between the two.

12. (a) What is the $ax^2 + bx + c$ form good for?

    (b) What is the $p(x+q)^2 + r$ form good for?

To figure out how to go back and forth, let's start with some simple examples.

13. Let $f(x) = (x+4)^2 + 2$. Write f in the form $p(x+q)^2 + r$.

14. Let $f(x) = (x - \frac{1}{2})^2 - 4$. Write f in the form $p(x+q)^2 + r$.

Hopefully, that was easy. Going the other way is not quite as easy. But look at your previous two answers, and compare some things.

15. What is p in these examples?

15. ______________

16. What's the relationship between b and q?

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17. Let $f(x) = x^2 + 6x + 5$.
    (a) If you were going to write f in the form $p(x + q)^2 + r$, what should q be?

    (a) ______________

    (b) What should p be?

    (b) ______________

    (c) There's only one value left to figure out, r. Can you do that? (You know p and you know q, so you might be able to set up an equation where r is the only variable...)

18. Try another one. Let $f(x) = x^2 - 4x + 1$.
    (a) Notice the sign in front of b. How does that affect things? What should q be?

    (a) ______________

    (b) What should p be?

    (b) ______________

    (c) Can you find r?

In all the previous cases, both a and p were equal to 1. That will no longer be the case.

19. Let $f(x) = 3(x + 4)^2 + 2$. Write f in the form $ax^2 + bx + c$.

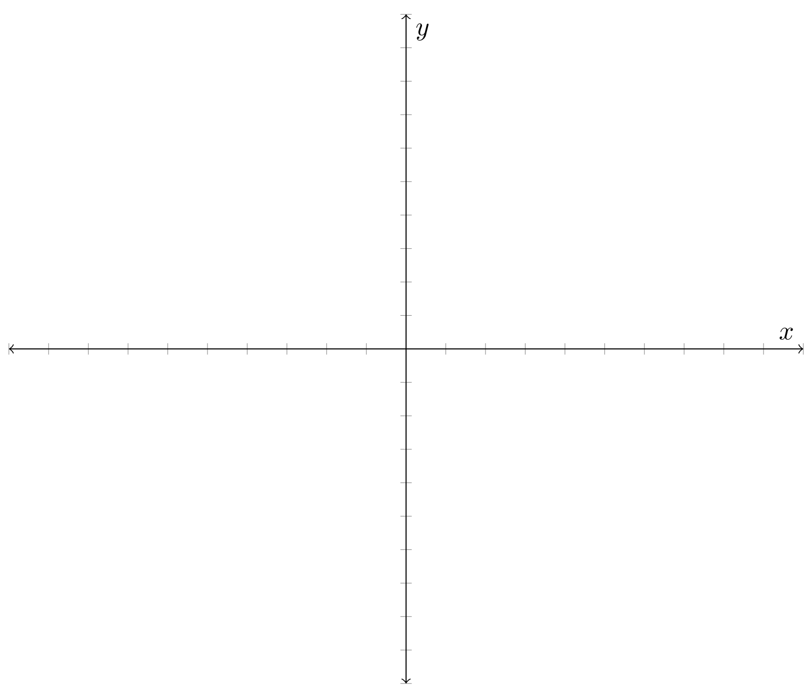
20. Let $f(x) = -(x - \frac{1}{2})^2 - 4$. Write f in the form $ax^2 + bx + c$.

21. What's the relationship between a and p?

22. Do you think the relationship between b and q will remain the same as it was in Questions 17 and 18? Why or why not?

23. The cases where $a = p = 1$ were so easy, right? (Hopefully....) Let's say you have $g(x) = 2x^2 - 8x + 2$. Can you find a way to rewrite g so that you can use the methods from those previous questions? Do that, and try to write g in the form $p(x+q)^2 + r$.

24. Try it now with $h(x) = 3x^2 + 6x + 2$. Your numbers won't be as nice, but try to just copy the steps you've used previously. Then sketch a graph of h.



25. Write out your method that you can use in general to take something in the form $ax^2 + bx + c$ and write it in the form $p(x+q)^2 + r$. Write in a way that makes sense to you.